

Delhi School of Economics
Department of Economics

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Entrance Examination for M. A. Economics
Option A (Series 02)
June 26, 2004

Time. 3 hours

Maximum marks. 100

General instructions. Please read the following instructions carefully.

- Check that you have a bubble-sheet accompanying this examination booklet. Do not break the seal on this booklet until instructed to do so by the invigilator.
- Immediately on receipt of this booklet, fill in your Name and Roll Number on the small slip attached to this booklet. Do not write this information anywhere else on this booklet.
- Following the instructions on the bubble-sheet, fill in the required information in Boxes 1, 2, 4, 5 and 6 on the bubble-sheet. The invigilator will sign in Box 3.
- This examination has two parts, 1 and 2. Part 1 is to be answered on the bubble-sheet, while Part 2 is to be answered in the blank space provided on this booklet itself.
- When you finish, hand in this booklet and the bubble-sheet to the invigilator.
- Do not disturb your neighbours at any time. Anyone engaging in illegal examination practices will be immediately evicted and that person's candidature will be cancelled.

Do not write below this line.

This space is for official use only.

Marks tally

Question	Marks
1	
2	
3	
4	
Total	

Write your Roll No. and Name on this slip and not on the answer book

Roll No. _____

Name: _____

Part 1

Instructions.

- Check that this booklet has pages 1 through 26. Also check that the bottom of each page is marked with *EEE 2004 A 02*.
- This part of the examination consists of 30 multiple-choice questions. Each question is followed by four possible answers, one of which is correct. Indicate the correct answer on the bubble-sheet, not on this booklet.
- Each correct choice will earn you 2 marks. However, you will lose 1 mark for each incorrect choice. If you shade none of the bubbles or more than one bubble, you will get 0 for that question.
- Use pages 22 through 26 of this booklet, marked **Rough work** to do your calculations, drawings, etc. Your "Rough work" will not be read or checked.
- This part of the examination will be checked by a machine. Therefore, it is very important that you follow the instructions on the bubble-sheet.

You may begin now. Good luck!

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1. Consider sets A , B and C , and the following propositions: (i) If $(A \cup B) \cap (A \cup C) = \emptyset$, then $B \cap C = \emptyset$. (ii) If $(A \cap B) \cup (A \cap C) = \emptyset$, then $B \cap C = \emptyset$.
 - (a) Both propositions are true
 - (b) Only proposition (i) is true
 - (c) Only proposition (ii) is true
 - (d) Both propositions are false
 2. Suppose an astronomer asserts "Every solar system in the Milky Way galaxy contains a planet such that all the moons revolving around that planet have life forms." If this assertion is false, which of the following assertions must be true?
 - (a) Every solar system in the Milky Way galaxy contains a planet such that none of the moons revolving around that planet have life forms.
 - (b) There exists a solar system in the Milky Way galaxy such that every planet in it has a moon revolving around it without any life forms.
 - (c) There exists a solar system in the Milky Way galaxy with a planet such that all the moons revolving around it have no life forms.
 - (d) There exists a solar system in the Milky Way galaxy such that every moon revolving around every planet in it has no life forms.

Situation I. Answer 3 and 4 on the basis of the following information. Consider a competitive exchange economy with two agents (1 and 2) and two goods (X and Y). Agent 1's endowment of (X, Y) is $(1000, 0)$ and Agent 2's endowment of (X, Y) is $(0, 1000)$. An allocation for Agent i is denoted (x_i, y_i) , where x_i is his allocation of X and y_i is his allocation of Y . Agent i 's objective is to choose allocations (x_i, y_i) to maximize his utility $\min\{x_i, y_i\}$.

3. The set of all Pareto efficient allocations for this economy consists of allocations of the form (x_1, y_1) and (x_2, y_2) , where $(x_2, y_2) = (1000 - x_1, 1000 - y_1)$ and

- (a) $x_1 = y_1 = 500$
- (b) $(x_1, y_1) = (1000, 0)$ and $(x_1, y_1) = (0, 1000)$
- (c) $x_1 = y_1$, for $x_1 \in [0, 1000]$
- (d) $x_1 \in [0, 1000]$ and $y_1 \in [0, 1000]$

4. The set of all competitive equilibrium allocations for this economy consists of allocations of the form (x_1, y_1) and (x_2, y_2) , where $(x_2, y_2) = (1000 - x_1, 1000 - y_1)$ and

- (a) $x_1 = y_1 = 500$
- (b) $(x_1, y_1) = (1000, 0)$ and $(x_1, y_1) = (0, 1000)$
- (c) $x_1 = y_1$, for $x_1 \in [0, 1000]$
- (d) $x_1 \in [0, 1000]$ and $y_1 \in [0, 1000]$

Situation II. Answer 5, 6 and 7 on the basis of the following information. Consider the exchange economy of Situation I with one change only: Agent 1's endowment of (X, Y) is $(999, 0)$.

5. Consider allocation α in which $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (999, 999)$, and allocation β in which $(x_1, y_1) = (500, 500)$ and $(x_2, y_2) = (499, 500)$.

- (a) α is Pareto efficient but β is not
- (b) β is Pareto efficient but α is not
- (c) Neither α , nor β is Pareto efficient
- (d) Both α and β are Pareto efficient

6. The set of all competitive equilibrium allocations for this economy consists of allocations of the form (x_1, y_1) and (x_2, y_2) , where $(x_2, y_2) = (999 - x_1, 1000 - y_1)$ and

- (a) $x_1 = 0$ and $y_1 \in [0, 1]$
- (b) $x_1 = 999$ and $y_1 \in [999, 1000]$
- (c) $x_1 \in [0, 999]$ and $y_1 \in [0, 1000]$
- (d) $x_1 = 499$ and $y_1 = 500$

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7. Assuming the sum of prices is 1, the competitive equilibrium prices (p_1, p_2) are
- (a) $(1, 0)$
 - (b) $(0, 1)$
 - (c) $(1/2, 1/2)$
 - (d) $(500/999, 499/999)$

Situation III. Answer 8, 9, 10, 11 and 12 on the basis of the following information. Consider a Cournot duopoly with inverse market demand function $p = 1 - q_1 - q_2$, where p is the market price and q_i is firm i 's output. Suppose Firm 1's cost function is $C_1(q_1) = cq_1$, where $c \in [0, 3/4]$, and Firm 2's cost function is $C_2(q_2) = q_2^2/2$.

8. In Cournot equilibrium, if Firm i 's market-share is defined by $s_i = q_i/(q_1 + q_2)$, then
- (a) $s_1 = (2 - 2c)/(3 - 4c)$ and $s_2 = (1 - 2c)/(3 - 4c)$
 - (b) $s_1 = 2c/(3 - 2c)$ and $s_2 = (3 - 4c)/(3 - 2c)$
 - (c) $s_1 = (3 - 4c)/(3 - 2c)$ and $s_2 = 2c/(3 - 2c)$
 - (d) $s_1 = (1 - 2c)/(3 - 4c)$ and $s_2 = (2 - 2c)/(3 - 4c)$
9. Is the derived Cournot outcome Pareto efficient from the point-of-view of the two firms?
- (a) Yes
 - (b) No
 - (c) This is undecidable in the Arrow-Debreu model
 - (d) Cannot be determined if prices are sticky
10. Suppose, prior to the choice of quantities by the two firms, Firm 1 can choose $\hat{c}$ by investing $(9\hat{c}^2 - 13\hat{c})/18$. Assuming that the firm chooses $\hat{c}$ anticipating Cournot quantity competition thereafter, the optimal choice of $\hat{c}$ is
- (a) $1/2$
 - (b) $2/3$
 - (c) $3/4$
 - (d) 0
11. Suppose Firm 1 takes-over Firm 2 to become a monopolist and $c = 2/3$. The new monopolist firm will produce
- (a) $2/3$ of the output in Plant 1 and $1/3$ of the output in Plant 2
 - (b) $1/3$ of the output in Plant 1 and $2/3$ of the output in Plant 2
 - (c) Divide the output equally between the two plants
 - (d) Produce nothing in Plant 1

12. Suppose $c = 1/2$ and Firm 1 can credibly commit itself to producing any output level q_1 that it publicly announces. What announcement q_1 will it make?

- (a) $1/2$
- (b) $1/3$
- (c) $1/4$
- (d) $1/5$

Situation IV. Answer 13, 14, 15, 16 and 17 on the basis of the following information. Consider a person who consumes water and bread, deriving utility xy if x is the amount of water consumed and y is the amount of bread consumed. Suppose this person's income is Rs. 10, the unit price of bread is Rs. 3 and the unit price of water facing this person is Re. 1. The price of water incorporates a per unit subsidy of Re. 1, i.e., for every unit of water consumed by this person, she pays Re. 1 to the water supplier and the government pays Re. 1 to the water supplier. Suppose this person's demand is (x_0, y_0) .

13. Suppose the water subsidy is removed and she has to pay Rs. 2 for each unit of water. Let (x_1, y_1) be the new demand. What is the resulting change in this person's demand, i.e., what is $(x_1 - x_0, y_1 - y_0)$?

- (a) $(-5/2, 0)$
- (b) $(-5/2, 10/9)$
- (c) $(-10/3, 10/9)$
- (d) $(-10/3, 0)$

14. If the government provides this person a lump-sum income subsidy that exactly offsets her utility loss on account of removal of the water subsidy, then the required lump-sum subsidy is

- (a) $9/4$
- (b) $5/3$
- (c) $\sqrt{56} + 4$
- (d) $\sqrt{200} - 10$

15. The income effect of the removal of the water subsidy is

- (a) $\left(\frac{10+\sqrt{200}}{4}, \frac{10+\sqrt{200}}{6}\right)$
- (b) $\left(\frac{10-\sqrt{200}}{4}, \frac{10-\sqrt{200}}{6}\right)$
- (c) $(-1/2, -1/3)$
- (d) $\left(\frac{5-\sqrt{200}}{4}, \frac{15-\sqrt{200}}{6}\right)$

16. The effect on the government's budget of the replacement of the water subsidy by the compensating income subsidy is

- (a) Deficit decreases by $15 - \sqrt{200}$
- (b) Deficit decreases by $1 - \sqrt{56}$
- (c) Deficit decreases by $10/3$
- (d) Deficit decreases by $11/4$

17. Assuming that government funds have a positive opportunity cost, the government can make a Pareto improvement by

- (a) removing the water subsidy
- (b) increasing the water subsidy
- (c) imposing a water tax and providing a lump-sum subsidy
- (d) removing the water subsidy and providing a lump-sum subsidy

V. Answer 18 and 19 on the basis of the following information. Suppose Y is aggregate real income, r is the real rate of interest, M is the exogenously given nominal money stock, P is the aggregate price level and G is real government expenditure. Consumption demand is $cY + \beta M/P$ and investment demand is α/r . The transactions demand for money balances is λY and the speculative demand for money balances is μ/r . Suppose Y and r are endogenously determined variables, while $c \in (0, 1)$, $\beta > 0$, $\alpha > 0$, $\lambda > 0$ and $\mu > 0$ are exogenously given parameters.

18. The rate of change of aggregate demand with respect to expansionary monetary policy is

- (a) $(\mu\beta + \alpha)/P[(1 - c) + \alpha\lambda]$
- (b) $(\mu\beta + \alpha)/P[\mu(1 - c) + \alpha\lambda]$
- (c) $(\beta + \alpha)/P[\mu(1 - c) + \alpha\lambda]$
- (d) $(\mu\beta + \alpha)/P[(1 - c) + \alpha\lambda]$

19. The rate of change of aggregate demand with respect to expansionary fiscal policy is

- (a) $\mu/[(1 - c) + \alpha\lambda]$
- (b) $\mu/[(1 - c) + \mu\alpha\lambda]$
- (c) $1/[\mu(1 - c) + \alpha\lambda]$
- (d) $1/[(1 - c) + \mu\alpha\lambda]$

VI. Answer 20 on the basis of the following information. Let $w = W/P$ be the real wage rate, where W is the nominal wage rate and P is the aggregate price level. The demand for labour is given by $D(w) = 1 - w$ and the supply of labour is described by the equation $S(w) = w$. Suppose that, if N is the employment level, then $f(N)$ is the aggregate output.

20. If nominal wage is always such that the labour market clears, then the aggregate supply curve is given by the equation

- (a) $Y = Pf(N)$
- (b) $Y = f(N)$
- (c) $Y = Pf(1/2)$
- (d) $Y = f(1/2)$

VII. Answer 21, 22 and 23 on the basis of the following information. Consider the labour market of Situation VI with the following change. Suppose the nominal wage rate W is always such that it minimizes $|D(W/P) - S(W/P)|$ subject to the constraint $W \geq W_0$, where $W_0 > 0$ is an exogenously given minimum nominal wage.

21. Given the price level P , the nominal wage W is

- (a) $\min\{W_0, P/2\}$
- (b) $\max\{W_0, P/2\}$
- (c) $1/2$
- (d) $W_0/2$

22. Given the price level P , the employment level is

- (a) $\min\{1/2, 1 - W_0/P\}$
- (b) $\max\{1/2, 1 - W_0/P\}$
- (c) $1 - W_0/2P$
- (d) $1 - 1/2P$

23. Given the price level P such that $1/2 \leq 1 - W_0/P$, the aggregate supply is

- (a) $f(1 - W_0/P)$
- (b) $f(1 - W_0/2P)$
- (c) $f(1/2)$
- (d) $f(1 - 1/2P)$

VIII. Answer 24 and 25 on the basis of the following information. Consider the economy described in Situations V and VII.

24. The effects of an expansionary monetary policy on equilibrium output Y and equilibrium price level P are

- (a) both Y and P increase
- (b) Y increases and P does not decrease
- (c) Y does not decrease and P increases
- (d) Y and P are unchanged

25. The effects of a contractionary fiscal policy on equilibrium output Y and equilibrium price level P are

- (a) both Y and P decrease
- (b) Y does not increase and P decreases
- (c) Y decreases and P does not increase
- (d) Y and P are unchanged

26. Consider an examinee answering a multiple-choice examination. For a particular question with 4 choices, the probability that the examinee knows the answer is $1/3$, the probability that she makes the correct choice given that she knows the answer is 1, and the probability that she makes the correct choice given that she does not know the answer is $1/4$. The probability that she knew the answer given that she has made the correct choice is

- (a) $1/3$
- (b) $3/4$
- (c) $2/3$
- (d) $5/6$

27. Each of four economists is asked to prescribe one out of four economic policies. Each economist is equally likely to prescribe any of the four different policies. What is the probability that each of the economists prescribes a different policy?

- (a) $3/8$
- (b) $1/256$
- (c) $3/16$
- (d) $3/32$

28. There are three identical boxes, each with two drawers. Box A contains a gold coin in each drawer. Box B contains a silver coin in each drawer. Box C contains a gold coin in one drawer and a silver coin in another drawer. A box is chosen, a drawer opened and a gold coin is found. What is the probability that the chosen box is A ?

- (a) $2/3$
- (b) $1/2$
- (c) $1/3$
- (d) $3/4$

29. 10 outcomes of a random variable were recorded. The sample mean is 0 and the sample variance is 4. It is discovered that two outcomes were recorded incorrectly: one outcome was recorded as -5 instead of -6 and another outcome was recorded as 11 instead of 12. What is the correct variance?

- (a) 4
- (b) 3.6
- (c) 7.4
- (d) 5.2

30. Suppose X and Y are independent random variables with standard Normal distributions. The probability of $X < -1$ is some $p \in (0, 1)$. What is the probability of the event: $X^2 > 1$ and $Y^2 > 1$?

- (a) $2p$
- (b) $4p$
- (c) $4p^2$
- (d) $2p^2$

Part 2

Instructions. This part of the examination consists of 4 questions. Each question is worth 10 marks; so budget approximately 18 minutes for each question. Answer each question in the space provided after that question. Do not use the space for Rough Work to write your answers; the Rough Work will not be read. Marks for each sub-part of a question are indicated at the end of that sub-part.

Question 1. Answer either (A) or (B). Each part is worth 10 marks.

(A) Consider a closed economy with aggregate output Y , rate of interest r , lump-sum tax T , government expenditure G , nominal money supply M and price level P . For this economy, the consumption function is $C(Y - T)$, the investment function is $I(r)$ and the demand-for-money function is $L(Y, r)$.

Suppose M and G are exogenously given, and the law requires the government to balance its budget. What is the effect of a marginal increase in G on aggregate demand? Explain the expansionary effect of such an increment.

(B) In a model with flexible exchange rates and perfect capital mobility, examine the effects of an increase in the foreign price level.

Answer.

Question 2. Answer either (A) or (B). Each part is worth 10 marks.

(A) Suppose n friends are trying to decide who will pay for everyone's dinner. They decide to choose this person (the "loser") by playing "odd man out". Each person is given a coin that has probability p of falling Heads. Each person tosses his coin. A person is said to be the loser if his toss outcome is different from that of the other $n - 1$ persons' tosses.

(a) If this game is played exactly once, what is the probability of finding a loser?

(b) Suppose the friends repeat this game until there is a loser. What is the probability that the game will be repeated exactly k times?

(B) A continuous random variable X takes values in the interval $[2, 8]$. Suppose $\text{Prob}(X \leq c) = \int_{-\infty}^c a(x+3) dx$, where $c \in (-\infty, \infty)$.

(a) Calculate a .

(b) Calculate $\text{Prob}(3 < X < 5)$.

(c) Calculate $\text{Prob}(X \geq 4)$.

Answer.

Question 3. Answer any two out of (A), (B) and (C). Each part is worth 5 marks.

(A) Consider a production function given by $(x_1, x_2) \mapsto f(x_1, x_2)$, where x_1 and x_2 are the quantities of the two inputs and $f(x_1, x_2)$ is the resulting output. Suppose

(a) f has the constant-returns-to-scale property, i.e., $f(tx_1, tx_2) = tf(x_1, x_2)$ for every $t > 0$, and

(b) the marginal product of the first input is always decreasing, i.e., $\partial^2 f / \partial x_1^2 < 0$.

Show that the marginal product of either input increases with an increase in the use of the other input.

(B) Suppose the monsoon can be Good, Average or Bad, which we label State 1, State 2 and State 3 respectively. Suppose three securities (1, 2 and 3) are traded on financial markets. Security i is described by a vector $a^i = (a_{i1}, a_{i2}, a_{i3})$, where a_{ij} refers to the value of Security i in State j . Suppose the three traded securities are $a^1 = (2, 1, -3)$, $a^2 = (0, 3, -1)$ and $a^3 = (-2, -1, a_{33})$.

Consider another security, described by $b = (b_1, b_2, b_3)$. For what values of a_{33} will it be the case that, for every $b = (b_1, b_2, b_3)$, there exists (x_1, x_2, x_3) such that

$$b = x_1 a^1 + x_2 a^2 + x_3 a^3 \quad (*)$$

Given such a value of a_{33} , provide an economic interpretation of equation (*).

(C) A set $X \subset \mathbb{R}^2$ is said to be convex if $tx + (1-t)y \in X$ for every $x \in X$, every $y \in X$ and every $t \in (0, 1)$. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is said to be convex if $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$ for every $t \in (0, 1)$.

Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is convex if and only if the set

$$\{(x, r) \in \mathbb{R}^2 \mid f(x) \geq r\}$$

is convex.

Answer.

Question 4. Answer either (A) or (B). Each part is worth 10 marks.

(A) A profit-maximizing firm produces electricity to meet the electricity demand of a city. The regulator has fixed the price of electricity and requires the firm to meet the entire demand at that price. Given this price, the city daily demands 4 units of electricity in Period 1 (6 AM and 6 PM) and 3 units of electricity in Period 2 (6 PM and 6 AM). Let z_i be the firm's electricity output in Period i . Electricity is produced using a fixed input x (say, plant) and a variable input y (say, fuel). The plant is invariant across periods, but the firm can employ different amounts of fuel in the two periods, say y_1 and y_2 . In Period i , the relationship between the inputs and the output is $z_i = \sqrt{xy_i}$.

- (a) Derive the firm's short-run cost function for a given period.
- (b) Derive the firm's long-run cost function for a given period.
- (c) Given that the firm must obey the regulator, what plant size x will the firm choose?
- (d) Does the answer to (c) change if the electricity demand changes to 3.5 units in each period? If so, what is the new plant size?

(B) There is a selection committee with three members (1, 2 and 3) who have to choose one of four candidates (a, b, c and d). The preferences of the members among the candidates are as follows

$$u_1(d) < u_1(c) < u_1(b) < u_1(a)$$

$$u_2(c) < u_2(d) < u_2(a) < u_2(b)$$

$$u_3(c) < u_3(a) < u_3(b) < u_3(d)$$

where u_i is the utility function of person i . The following procedure is used to choose among the candidates. First, Member 1 vetoes one of the candidates, thereby knocking him out of contention. Then Member 2 vetoes one of the three remaining candidates, thereby knocking him out of contention. Finally Member 3 vetoes one of the two remaining candidates. The candidate not vetoed by any of the members is chosen. Each member wishes to maximize his utility.

Who will Member 1 veto? Who will Member 2 veto? Who will Member 3 veto? Justify your answers.

(Hint: From the point of view of any particular committee member, maximization of his utility requires him to look forward and predict the behaviour of members who will exercise their veto *after* that particular committee member.)

Answer.